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SHORTER COMMUNICATIONS

DISPLACEMENT-THICKNESS INDUCED PRESSURES ON A FLAT PLATE WITH HOMOGENEOUS AND HETEROGENEOUS VECTORED INJECTION

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NOMENCLATURE

c_f, mass fraction of species f, ρ_f/ρ;
C, coefficient of the linear viscosity-temperature relation;
C_F, local skin-friction coefficient, $\tau_w/\frac{1}{2}\rho_e u_e^2$;
C_p, C_{pf}, specific heats at constant pressure of mixture and species f respectively;
C_v, C_{vf}, specific heats at constant volume of mixture and species f respectively;
D_{fi}, binary diffusion coefficient;
f, dimensionless stream function, $\Psi/\sqrt{2\xi}$;
g, dimensionless total enthalpy ratio, H/H_e;
h, h_f, static enthalpies of mixture and species f respectively;
k, k_f, thermal conductivities of mixture and species f respectively;
L, length of plate (used as reference length);
Le, Lewis number, $\rho D_{fi} C_p/k$;
M_∞, flow Mach number;
M_f, molecular weight of species f;
$\dot{m}_w$, surface mass transfer rate;
p, thermodynamic pressure;
Pr, Prandtl number $C_p\mu/k$;
q_w, local heat transfer rate per unit area, $-[k(\partial T/\partial y)+(C_{pf}-C_{pi})T\{\rho D_{fi}(\partial c_f/\partial y)\}]_w$;
R', universal gas constant;
Re_x, Reynolds number, $\rho_e u_e x/\mu_e$;
T, temperature;
u, v, component of velocity parallel and normal to surface, respectively;
x, y, coordinates along and normal to the body, respectively.

Greek symbols

α, parameter defined as $(C_{pf}/C_p)[\{c_f(M_i-M_f) + M_f\}/M_i]$;
β, pressure gradient parameter, $(2H_e/u_e h_e)(du_e/d\xi)$;
γ, ratio of specific heats;
Δ, δ^*, effective displacement thickness and displacement thickness respectively;
η, transformed y-coordinate, $(u_e/\sqrt{2\xi})\int_0^y \rho\,dy$;
θ, angle of injection (as shown in Fig. 1);
μ, μ_f, viscosities of mixture and species f respectively;
ξ, transformed x-coordinate, $\int_0^x \rho_e u_e \mu_e\,dx$;
ξ^*, normalized ξ, ξ/ξ_L;
ρ, density of mixture;
τ_w, wall shear stress, $\mu_w(\partial u/\partial y)_w$;
$\bar{\chi}$, hypersonic interaction parameter, $M_e^3(C/Re_x)^{1/2}$.

Subscripts

e, evaluated at edge of the boundary layer;
f, evaluated for species f (air);
i, evaluated for species i (injected species);
L, evaluated at trailing end of plate;
o, evaluated with no injection;
w, evaluated at the surface;
∞, conditions ahead of the shock wave;
Prime indicates differentiation with respect to η;
Symbols with local reference are defined in text.

INTRODUCTION

THE SURFACE injection presents a possible means of alleviating the aerodynamic heating problem which assumes formidable dimensions at hypersonic flight speeds. In addition, the use of injection is also made for controlling the boundary layer and inducing the control forces on aerodynamic vehicles. The surface injection of a gas materially influences the processes occurring within the boundary layer of an object moving through the atmosphere. Most of the earlier works[1, 2] connected with the study of surface mass transfer are pertinent to vertical heterogeneous surface injection and without the study of the corresponding displacement-

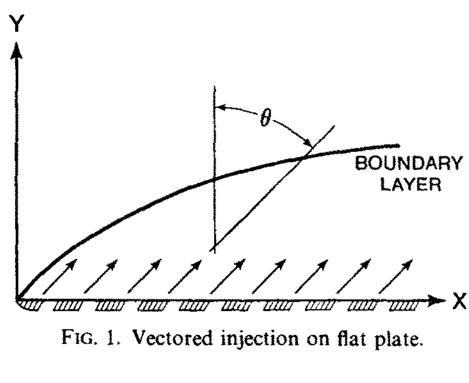

FIG. 1. Vectored injection on flat plate.

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thickness induced pressures. Under the hypersonic flight conditions, the displacement thickness growth is substantial and the pressures induced by its growth are greatly increased with surface mass transfer. The present investigation serves a two-fold purpose: (a) it examines for the first time the effect of vectored heterogeneous injection in a flow field with pressure gradient and (b) it gives first-order approximation for the problem of strong-interaction flows which may not be treated under the category of similar solutions [3]. The effects of vectored air (homogeneous) injection in the 'weak-to-moderate' interaction region have been investigated in Ref. [4]. They treated the interaction effect as a small, non-similar perturbation caused by the self-induced pressure gradient on a basic self-similar isobaric flow solution with heat transfer and vectored air injection.

GOVERNING EQUATIONS AND SOLUTION PROCEDURE

The laminar boundary-layer equations for a binary gas mixture of perfect gases with no thermal diffusion and chemical reactions for two-dimensional high-speed flow over a flat plate are used as the governing equations. These equations written in a form incorporating the Levy–Lees transformations and appropriate hypersonic assumptions and suitable for the flows with a pressure gradient are

momentum:

$$(CF'')' = -\alpha\beta[G + 1 - (F' + 1)^2] - (F + \eta)F'' + 2\xi^*\left\{(F' + 1)\frac{\partial F'}{\partial \xi^*} - F''\left(\frac{\partial F}{\partial \xi^*}\right)\right\}; \quad (1)$$

energy:

$$\pi' = -(F + \eta)G' + 2\xi^* \times \left\{(F' + 1)\frac{\partial G}{\partial \xi^*} - G'\frac{\partial F}{\partial \xi^*}\right\}; \quad (2)$$

species:

$$\phi' = -(F + \eta)c_f' + 2\xi^* \times \left\{(F' + 1)\frac{\partial c_f}{\partial \xi^*} - c_f'\frac{\partial F}{\partial \xi^*}\right\}, \quad (3)$$

where

$$F' = f' - 1, \quad G = g - 1, \quad (4)$$

$$\pi = \frac{C}{Pr}\left\{G' + 2(Pr - 1)F''(F' + 1) + \left(\frac{C_{pf} - C_{pi}}{C_p}\right) \times (Le - 1)[(G + 1) - (F' + 1)^2]C_f'\right\} \quad (5)$$

and

$$\phi = \frac{CLe}{Pr}C_f'. \quad (6)$$

The boundary conditions associated with the problem are:

(i) At the wall of known surface temperature with both vertical and tangential injection, continuum boundary conditions are used

$$f_w = -\Omega\cos\theta(Re_x/2)^{1/2}, \quad f_w' = (\rho_e/\rho_w)\Omega\sin\theta,$$

$$g_w = (C_{pw}, T_w/H_e) + (u_e^2/2H_e)f_w'^2,$$

$c_{f_w} = c_f(\eta) = 1$ for no injection of foreign gas, or, with injection the Eckert's condition is employed, namely,

$$c_{f_w}' = \Omega\cos\theta(\rho_e/\rho_w)(Pr/\mu Le)_w(2\xi)^{1/2}c_{f_w}. \quad (7)$$

(ii) In the free stream the boundary conditions employed are

$$f' = g = c_f = 1. \quad (8)$$

Equations (1–3) with the associated boundary conditions given by the expressions (7) and (8) are solved using the method developed by Smith and Jaffe [5] and applied to a number of boundary layer problems. This method, sometimes referred to as the 'difference–differential' technique, consists of replacing the partial derivatives with respect to the transformed streamwise direction by a backward finite-difference scheme while retaining the derivatives in the direction normal to the surface in the governing equations. The partial differential equations are thus approximated by ordinary differential equations that are solved simultaneously at a given station as the calculation proceeds downstream. The details of the method are provided in Ref. [5]. To start the solution, however, a different approach is adopted. The usual practice is to specify initial values of F, G and C_f at $x = 0$ (ie $\xi^* = 0$). Since all the streamwise derivatives in equations (1–3) are multiplied by ξ^*, these equations reduce to ordinary differential equations in η at $\xi^* = 0$. The solutions of these ordinary differential equations are then used as the starting values. However, as pointed out by Rodkiewicz [6], this approach is questionable because it assumes the validity of the boundary layer equations at $x = 0$. Therefore, a 'correct' approach would be to start the solutions from a line $\xi^* = \xi_1^*$ very close to the leading edge and use the series expansion solutions [7] as the starting values on this line. A value of $\xi_1^* = 10^{-3}$ corresponding to $\bar{\chi} \sim O(10^2)$ is used at the starting station which is assumed to have a solid surface. Thus, $C_f(\eta) = 1$ is the solution to equation (3) at this station. As the solution proceeds downstream, the spacing $\Delta\xi^*$ between the stations is increased in a geometric progression. Expressions for the various fluid properties appearing in equations (1–3) are obtained from Ref. [8].

UTILIZATION OF RESULTS AND CONCLUDING REMARKS

The solutions to the governing equations (1–3) yield profiles of the velocity, temperature and species concentration. These are utilized to compute local skin-friction coefficient, surface heat transfer and displacement thickness. A first-order contribution to the displacement-thickness induced pressure due to the surface mass-transfer effects is also evaluated using the tangent-wedge approximation. The effective body shape Δ for computing the induced pressures is obtained from the relation

$$\frac{d\Delta}{dx} \simeq \frac{d\delta^*}{dx} + \frac{\rho_w v_w}{\rho_e u_e} \simeq \frac{d\delta^*}{dx} + \Omega\cos\theta. \quad (9)$$

In tangent-wedge approximation, the pressure at any point on a slender body of thickness Δ is approximated by the pressure across an oblique shock that produces the local flow deflection $d\Delta/dx$. For a finite product $K_s(x) = M_e\, d\Delta/dx$, this approximation is [9]

$$\frac{p_e}{p_\infty} = 1 + \gamma K_s\left[\frac{\gamma + 1}{4}K_s + \left\{\left(\frac{\gamma + 1}{4}K_s\right)^2 + 1\right\}^{1/2}\right] + O(M_e^{-2}) \quad (10)$$

Expression (10) along with (9) gives the first-order contribution to induced pressure due to surface mass-transfer effects. The numerically obtained solutions to the governing equations have been used to compute the boundary layer displacement thickness δ^* and subsequently the induced pressures utilizing equation (10).

Figures 2–4 display results obtained with vectored mass transfer at various angles of injection for the same surface mass flux. Also displayed in these figures are results obtained with normal surface air injection for the same injection rate. Vectoring the injection results in a finite velocity at the surface in the x-direction since the surface is considered to be completely porous. This results in an additional injection-induced acceleration of the flow immediately next to the surface which tends to oppose the effects due to the normal

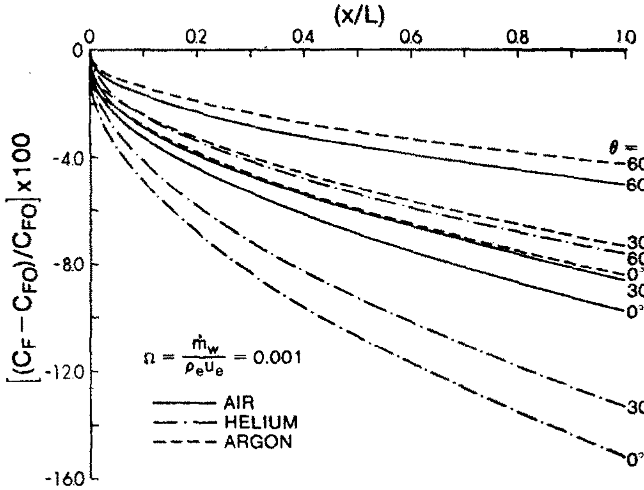

FIG. 2. Per cent changes in local skin-friction coefficient C_F with vectored injections of helium, argon and air; wall concentration specified by Eckert's condition.

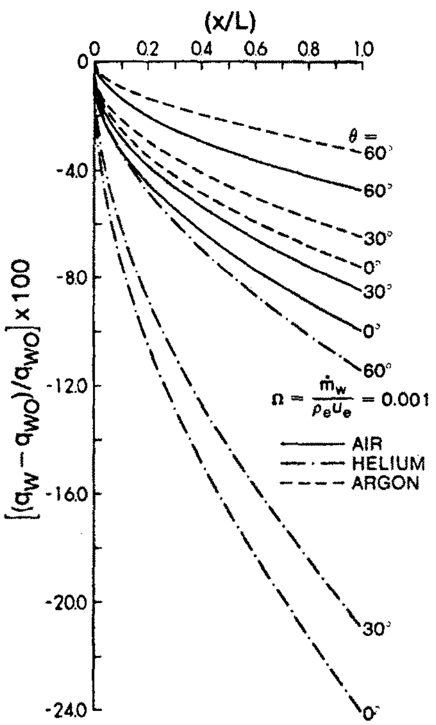

FIG. 3. Per cent changes in local surface heat transfer with vectored injections of helium, argon and air; wall concentration specified by Eckert's condition.

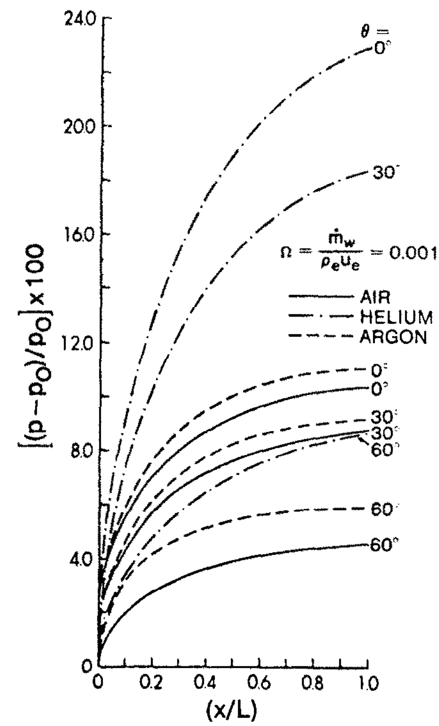

FIG. 4. Per cent changes in displacement-thickness induced pressures with vectored injections of helium, argon and air; wall concentration specified by Eckert's condition.

component of injection. Consequently, with increasing angles of injection, displacement thickness and induced pressures decrease, whereas skin-friction and wall heat transfer increase as compared to the normal injection case.

From the present results it may be said that helium appears to be a more effective coolant as compared to air or argon. However, the displacement-thickness induced pressures are highest with helium injection. It is noted that these large induced pressures may reduce the cooling effectiveness of helium, thereby increasing the relative merits of air as a coolant. Further, vectored injection presents a possible method of reducing displacement-thickness induced pressures in flows with surface mass transfer. Finally, these results which represent a first-order effect of surface mass transfer on induced pressures, may be utilized to recompute the flow-field for an interaction problem.

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NATURAL CONVECTIVE HEAT TRANSFER IN VERTICAL WAVY CHANNELS

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1. INTRODUCTION

ANALYSES of fluid flow over wavy walls have applications in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. In view of these applications Lekoudis *et al.* [1] have made a linear analysis of compressible boundary-layer flows over a wavy wall. Shankar and Sinha [2] have studied the effects of wall waviness on the well known Rayleigh problem. Lessen and Gangwani [3] have analysed the effect of small amplitude wall waviness upon the stability of the laminar boundary layer. In all these studies the authors have taken the wavy wall to be oriented in a horizontal direction and studied the effect of the waviness on the flow field.

The present authors, Vajravelu and Sastri [4] have made a systematic analysis of free convective heat transfer in a viscous fluid confined between a long vertical wavy wall and a parallel flat wall and have established that the flow and heat transfer characteristics are significantly affected by the wall waviness. The present problem is an extension of [4] for the case when the channel walls are wavy and is taken for study for two reasons: firstly, its solution will be useful in the stability analysis (the stability results will be presented in another paper) and secondly, the heat transfer results have a definite bearing on the design of oil or gas-fired boilers. Due consideration has been given to different cases of orientation of the channel walls (see Fig. 1), because any relative differences in the orientation can lead to significant changes in the heat transfer results. The governing equations have been solved exactly analogous to that of [4]. It is interesting, but not surprising, to note that the mean part of the solution coincides with that in [4], after modifications resulting from the different choices of the origin in [4] and in the present investigation, while the perturbed part of the solution is the contribution of the waviness of the walls.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Figure 1 depicts the various channels considered in this study. Let $Y = d + \varepsilon^* \cos KX$ ($=y_1$, say) and $Y = -d + \varepsilon^* \cos(KX + \omega)$ ($=y_2$, say) represent the channel walls, with ω taking values equal to 0, $\pi/2$, π and $3\pi/2$ to denote changes in the orientation of the channel walls, which are maintained at constant temperatures T_1 and T_2 respectively. Assuming the flow to be laminar, steady and two-dimensional the governing equations of the flow and heat transfer of the problem are exactly the same as those in [4]. Using the method of

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